XVIII. *On the Resolution of attractive Powers. By* Edward Waring, *M. D. F. R. S. and* Lucasian *Professor of Mathematics at* Cambridge.

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1. A FORCE acting at a given point may be resolved by an infinite number of ways into two, three, or more (n) forces acting at the same point, either in the same or different planes with the given force and each other; and, *vice versâ*, any number of such forces acting in the same or different planes may be reduced into one.

Ex. Fig. 1. Tab. III. Let a body A be acted on by three forces AB, AC, and AD, not being in the same plane; reduce any two of them AB and AC to one AE, by compleating the parallelogram ABEC; then reduce the two forces AE and AD to one AF by completing the parallelogram AEFD, and the three forces AB, AC, and AD, are reduced to the one AF.

2. If n forces act on the body A at the same time, and any $(n-1)$ of them be reduced to one, the force resulting will be situated in the same plane with the remaining, and force equivalent to the (n) forces.

3. If one force a be resolved into several others x, y, z, v, &c. situated in different planes, and the sines of the angles, which the forces y, z, v, &c. contain with the plane made by the direction of the forces x and a be respectively s, s', s'', &c. then will $sy \pm s'z \pm s''v \pm \text{\&c.} = 0$.

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PROBLEM I.

Fig. 2. Given the law of attraction of each of the parts of a given line in terms of their distance from a given point P; to find the attraction of the whole line ab on the point P.

Find the attraction of the line ab on the point P in the two directions Pf and fb by the following method. Draw Px from the point P to any point x of the line ab, the force acting on the point P by the particle xy will be the given function (determined from the given law of attraction) of the distance into the particle; draw also Ph perpendicular from the point P to the line ab, and let $Pf=a$, $hf=b$, and $fx=y$; then will the distance $Px=\surd(a^2\pm 2by+y^2)$, and the function of the distance into the particle $xy=\varphi\ (\surd(a^2\pm 2by+y^2))\times\dot{y}=F(y)\times\dot{y}$; let this be denoted by lx situated in the line Px, which resolve into two others $nx=\frac{y\dot{y}\times F:(y)}{Px=\surd(a^2\pm 2by+y^2)}$ situated in the line ab, and ln (in a direction parallel to $Pf)=\frac{a\dot{y}\times F:(y)}{\surd(a^2\pm 2by+y^2)}$; find the fluents of the fluxions $\frac{y\dot{y}\times F:(y)}{Px}$ and $\frac{a\dot{y}\times F:(y)}{Px}$ contained between the values af and fb of the line $fx=y$, which suppose Y and V respectively; through the point p draw Py parallel to $fb=Y$, and in the line Pf assume $Pu=V$; complete the parallelogram $Puzy$; Pz will be the force of the line ab on the point P.

Cor. If $F:(y)$ varies as any power or root ($2n$) of the distance $Px=\surd(a^2\pm 2by+y^2)$, and $n-\frac{1}{2}$ be an integer affirmative number or o, the fluents Y and V of both the fluxions can be found in finite algebraical terms of y; if $n-\frac{1}{2}$ be an integer negative number, both the fluents can be found in the above-

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mentioned finite terms together with the arc of a circle, whose radius is $\sqrt{a^2 - b^2}$ and tangent $y \mp b$, unless $n - \frac{1}{2} = -1$, in which case the fluent Y involves that circular arc, and also the logarithm of $y^2 \pm 2by + a^2$. If $n - \frac{1}{2}$ denotes a fraction whose denominator is 2, both the fluents can be expressed by the finite terms together with the log. of $y \pm b + \sqrt{(y^2 \pm 2by + a^2)}$. If the fluents be given, when n is a given quantity, and $n - \frac{1}{2}$ not a whole affirmative number, from them can be deduced the fluents of any fluxions resulting by increasing or diminishing n by a whole number, unless in the above-mentioned case of $n - \frac{1}{2} = -1$. If $b = 0$, and consequently the line Pf is perpendicular to the given line ab, the fluent Y will be expressed by the finite terms, unless $n - \frac{1}{2} = -1$, in which case it will be as $\frac{1}{2}$ log. $(y^2 + a^2)$ when properly corrected.

These fluxions $\dot{Y}$ and $\dot{V}$ may be transformed into others, whose variable quantity is $Px = u$ the distance from P, by substituting in the fluxions for y and $\dot{y}$ their respective values $\sqrt{(u^2 - a^2 + b^2)} \mp b$ and $\frac{u\dot{u}}{\sqrt{(u^2 - a^2 + b^2)}}$, and consequently for $\sqrt{(y^2 \pm 2by + a^2)}$ its value u.

PROBLEM II.

Fig. 3. Given the attraction of each of the parts of a given surface in terms of their distance from a given point P, and an equation expressing the relation between an abscis $Ap = x$, and its correspondent ordinates $pm = y$ of the surface; to find the attraction of the surface on the given point P.

First, by the preceding proposition find the attractions Y and V of any ordinate $m\,p\,m'$ in the directions of the ordinate pm and of the line Pp; and from the equation expressing the relation

relation between the abſciſs and ordinates of the given curve, find the abſciſs in terms of the ordinates $(pm) = \pi : (y)$, and thence $\dot{x} = \varphi : (y) \times \dot{y}$ and $\sqrt{(a'^2 \pm 2sa'x + x^2)} = \varphi' : (y)$, where $PA = a'$ and $s =$ coſine of the angle, which the abſciſs Ap makes with the line PA; then find the fluents of the three fluxions $\dot{x} \times Y = \dot{y} \times Y \times \varphi : (y)$, $\dot{x} \times \frac{V \times x}{\sqrt{(a'^2 \pm 2sa'x + x^2}} = \varphi : (y) \times \dot{y} \times \frac{\pi : (y)}{\varphi : (y)} \times V$ and $\dot{x} \times \frac{a'V}{\sqrt{(a'^2 \pm 2sa'x + x^2)}} = \dot{y} \times \frac{a'V}{\varphi' : (y)}$ contained between the values of y, which correſpond to the extreme values of x, which ſuppoſe Y', V', and Z; and draw through the point P the lines Py and Pz reſpectively parallel to the ordinates pm and to the abſciſs Ap and equal to $r \times Y'$ and V'; aſſume Pu in the line $(PA) = t \times Z$, r and t denoting the ſines of the angles, which the ordinates pm and line AP make with the abſciſs Ap: reduce theſe three forces Py, Pz, and Pu, to one Pf, and Pf will be the force of the ſurface on the point P.

Cor. 1. If for y and $\dot{y}$ be ſubſtituted their values in terms of x and $\dot{x}$, deduced from the equation expreſſing the relation between the abſciſs Ap and ordinate pm of the given curve, thence will be deduced the above-mentioned fluents Y, V, Y', V', and Z, in terms of x; and in the ſame manner, if for x and $\dot{x}$ be ſubſtituted in the fluxions or fluents reſulting their values $\sqrt{(u^2 - a'^2 + 1^2 a'^2)} \mp sa'$, and its fluxion, there will reſult the above-mentioned fluxions or fluents in terms of u the diſtance from the point P.

Cor. 2. Let the curve be a circle, of which A is the center, PA a line perpendicular to the plane of the circle, and the ordinate pm perpendicular to the abſciſs Ap; the forces on each ſide of the abſciſs Ap will be equal, and the force in the direc-

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tion of the abscifs Ap will be equal to that in the contrary direction; the force in the direction (PA) $= 4 \times \int \frac{a\dot{u}}{\sqrt{(u^2 - a^2)}} \times \int \frac{u\dot{y}}{\sqrt{u^2+y^2}} \times F : (\sqrt{(u^2+y^2)}) = W$, in which $F : (\sqrt{u^2+y^2})$ is the function of the distance, according to which the given force on the particles varies; the fluent $\int \frac{u\dot{y}}{\sqrt{(u^2+y^2)}} \times F : \sqrt{(u^2+y^2)}$ is contained between the values o and $\sqrt{(r^2+a^2-u^2)}$ of the quantity y, and the fluent W is contained between the values a and $\sqrt{(a^2+r^2)}$ of the quantity u, where $a = PA$ and r the radius of the circle; but the same force is $= 2 \times 3{,}14159$ &c. $\times \int a\dot{u} \times F : (u)$, where $F : (u)$ denotes the given function of the distance (u), and the fluent is contained between the values a and $\sqrt{a^2+r^2}$ of u.

PROBLEM III.

To find the attraction of a given solid on a given point P. Find the attraction of every parallel section on that point by the preceding problem, and multiply it into the correspondent fluxion of the first abscissa AP, and also find the fluent of the resulting fluxion, which, properly corrected, multiply into the sine of the angle, which the first abscissa makes with the parallel sections, and the product will be proportional to the attraction of the solid on the given point P.

2. Fig. 4. Let the solid ABCH be generated by the rotation of a given curve round its axis AB, which passes through the point attracted P, and this solid be supposed to consist of small evanescent solids, whose bases are the surfaces EF, ef, &c. of spheres, of which the center is P, and altitudes Ff, &c. the

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increments of the baſe AB contained between the two contiguous ſurfaces EF and ef: from the points E and e of the curve draw ED and ed perpendicular to the axis AB, and ES perpendicular to the arc Ee of the given curve at the point E, and meeting the axis AB in S; then will the evaneſcent ſolid $EFfe = p \times PE \times FD \times Ff = p \times FD \times PS \times Dd$ (becauſe $Ff = \frac{PS \times Dd}{PE}$) $= p \times \overline{\sqrt{(z^2+y^2)} - z} \times \overline{z\dot{z} \mp y\dot{y}}$, where z and y denote reſpectively the abſciſs PD, and its correſpondent ordinate DE of the given curve.

The increment of the attraction of the ſurface EF on the point P in the direction PD will be as the increment of the ſurface $(p \times PE \times Dd) \times \frac{PD}{PE} \times$ force of each particle $= p \times PD \times Dd \times$ given force of the particle; but the fluent of the fluxion $PD \times Dd$ contained between the points E and F is $= \frac{1}{2}PE^2 - \frac{1}{2}PD^2 = \frac{1}{2}ED^2$, whence the attraction of the evaneſcent ſolid EFfe is as $\frac{1}{2}p \times ED^2 \times Ff \times F : (\sqrt{x^2+y^2})$ force of each given particle at the diſtance $(PE = \sqrt{(x^2+y^2)}) = \frac{1}{2}p \times ED^2 \times \frac{PS}{PE} \times Dd \times F : (\sqrt{z^2+y^2}) = \frac{1}{2}py^2 \times \frac{z\dot{z} \mp y\dot{y}}{\sqrt{(z^2+y^2)}} \times F : (\sqrt{(z^2+y^2)})$; the fluent of which, properly corrected, is as the attraction of the ſolid on the point P; p denotes the circumference of a circle, whoſe radius is 1.

Cor. 1. The fluxion of this ſolid is $\frac{1}{2}py^2\dot{z} = \dot{Y}$ which deduced from the preceding principles $= p \times (\sqrt{(z^2+y^2)} - z) \times (z\dot{z} \mp y\dot{y}) = \dot{V}$, and conſequently their fluents between two values of z, which correſpond to two values of $y = 0$, will be equal to each other.

Cor.

Cor. 2. The increment of the attraction of this solid as given in this proposition $\frac{4}{3}p \times y^2 \times \frac{z\dot{z} \mp y\dot{y}}{\sqrt{(z^2+y^2)}} \times F : (\sqrt{(z^2+y^2)}) = \dot{U}$, but in the preceding proposition the force of a circle on the point $P = p \times \int a\dot{u} \times F : (u)$, where $u = \sqrt{(z^2+y^2)}$ and $a = z$, and y or u the only variable quantity contained in the fluxion; and consequently the fluxion of the attraction of the solid $p \times \dot{z} \int z \frac{y\dot{y}}{\sqrt{z^2+y^2}} \times F : ((z^2+y^2)^{\frac{1}{2}}) = \dot{W}$; therefore, if for the fluent of $\frac{zy\dot{y}}{\sqrt{(z^2+y^2)}} \times F : ((z^2+y^2)^{\frac{1}{2}})$ be substituted its fluent contained between the values a and the value of y, which in the given equation corresponds to z; then the fluents of $\dot{U}$ and $\dot{W}$ contained between the two values of z, which corresponds to two values of $y=0$, will be equal to each other.

The difference of the fluents of $\dot{Y}$ and $\dot{V}$, &c. contained between any other two values of z, can easily be deduced from the difference of two segments of spheres.

1. It may not be improper to remark in this place, that from different methods of finding the sum of quantities, the fluents of fluxions, the integrals of increments, &c. quantities may often be deduced equal, which otherwise cannot without some difficulty; of which instances are contained in the Meditationes, and I shall here subjoin one or two more to those already given in this Paper.

Ex. 1. Any curvilinear area ABC, &c. may be supposed to consist of evanescent areas $EFef$, of which the base EF is the arc of a circle, whose radius is $PE = \sqrt{(z^2+y^2)}$ and sine $ED = y$, and altitude Ff, and consequently the fluxion of the area $= Ff \times$ arc (A) of a circle whose radius is PE and sine ED =

$\frac{PS}{PE} \times \dot{z} \times A = \frac{z\dot{z} \mp y\dot{y}}{\surd(z^2+y^2)} \times A = \dot{V}$; the fluent of $\dot{V}$ contained between the two values of z which correfpond to two values of $(y) = 0$ will be equal to the fluent of $y\dot{z}$ contained between the fame two values of z.

Ex. 2. The attraction of any circular arc EF in the direction PD on a point P (P being the center of a circle, of which EF is an arc, and ED the fine of that arc) will be as ED × force at diftance PE = ED × F : (PE); for the attraction in the direction $PDl'F$ at the point x is as the increment of the arc $xy \times F : (PE) \times \frac{Pl}{Px}$ (xl and yl' being at right angles to PF) $= U \times \frac{Px}{xl} \times \frac{Pl}{Px} \times F : (PE) = \frac{U' \times Pl}{lx} \times F : (PE) = \frac{u\dot{u}}{\sqrt{PE^2 - u^2}} \times F : (PE)$, if $u = Pl$; and confequently the fluent of it is as $\sqrt{PE^2 - u^2} \times F : (PE) = ED \times F : (PE)$, and the attraction of the furface $EFef$ will be as $ED \times Ff \times F : (PE) = ED \times \dot{z} \times \frac{PS}{PE} \times F : (PE) = y \times \frac{z\dot{z} \mp y\dot{y}}{\surd(z^2+y^2)} \times F : ((z^2+y^2)^{\frac{1}{2}} = \dot{V}$; the attraction of the curve will alfo vary as $\int \dot{z} \int \frac{z\dot{u} \times F : ((u^2+z^2)^{\frac{1}{2}})}{(z^2+u^2)^{\frac{1}{2}}} = W$, in which the fluent of $\frac{z\dot{u} \times F : ((z^2+u^2)^{\frac{1}{2}})}{(z^2+u^2)^{\frac{1}{2}}}$ is contained between $u = 0$ and $u = y$; the fluents of $\dot{V}$ and $\dot{W}$ contained between two values of z, which correfpond to two values of $y = 0$, will be equal to each other.

2. From a fimilar method may be deduced equalities between other like fluents, for the curve may be fuppofed to confift of other fimilar curve furfaces equally as circles, and the folid of fimilar fegments of other folids equally as fpheres.

3. From the fame principles may innumerable feriefes equal to each other be deduced; for by different converging feriefes find

find the fum of the fame quantity or quantities, and there will refult feriefes equal to each other: for inftance (fig. 5), if the time of falling down the arcs AC and BC and their interpolations from the principles delivered in the Meditationes Analyticæ, of which the difference let be D; find the difference between the times of a body's falling through BC when it began to fall from A and from B by a feries proceeding according to the dimenfions of AB $=o'$ a fmall quantity; and find, by a feries of the fame kind, the time of falling through AB; the fum of thefe two feriefes will be equal to D. Similar propofitions may be deduced from fluxional equations.

4. In fome cafes the ratios of the times of bodies falling through fome particular diftances to each other may be eafily known; for inftance, let the force vary as the $m-1$ power of the diftance (x), and a be the diftance from which the body began to fall, then the velocity varies as $\sqrt{(a^m-x^m)}$, and the increment of the time as $\frac{\dot{x}}{\sqrt{(a^m-x^m)}}$; but if the parts of different curves are proportional, then will a, x, and $\dot{x}$ vary in the fame ratio as each other, and confequently the time through proportional parts of the diftance will vary as $a^{\frac{2-m}{2}}$; and if the bodies be refifted likewife by a force which varies as the $\frac{2m-2}{m}$ power of the velocities, then the times through proportional parts will vary as before, that is, as $a^{\frac{2-m}{2}}$, where a denotes the proportional diftances from the points where the forces and refiftances are equal.

PROBLEM IV.

1. Fig. 6. Given an equation expreffing the relation between the two abfciffæ $z = AP$ and $x = Pp$ and their correfpondent ordinates $y = pm$ of a folid, to find its folid contents contained between two values of its firft abfciffæ z. Affume z as an invariable quantity, and from the equation refulting find the fluent Z of $y\dot{x}$ contained between the extreme values of x or y; then find the fluent of $Z\dot{z}$ contained between the given values of z, and the fluent multiplied into the product of the fines of the angles, which the firft abfciffa makes with the plane of the ordinates and fecond abfcifs, and the fecond abfcifs makes with its correfpondent ordinates, will be the folid content required.

2. Fig. 7. Let the firft abfcifs z of a folid be perpendicular to the planes of the ordinates, and the fecond abfcifs $Pp = x$ perpendicular to the ordinates themfelves $pm = y$. Firft, affume the firft abfcifs as invariable, and find the increment of the arc $p'm = (\dot{x}^2 + \dot{y}^2)^{\frac{1}{2}}$, then affume the fecond abfcifs Pp as conftant, and let mu be the fluxion of the ordinate y or u, when the fluxion of the firft abfcifs is $\dot{z} = ul$, where ul is perpendicular to the plane of the ordinates $p'pm$, and l a point of the furface of the folid; draw uh perpendicular to the arc $p'm$, and fince ul is conftituted at right angles to the plane $pp'm$, lh will cut the arc $p'm$ at right angles; but $uh = \frac{um \times pp'}{p'm} = \frac{\dot{u}\dot{x}}{\sqrt{(\dot{x}^2 + \dot{y}^2)}}$, $lh = (hu^2 + lu^2)^{\frac{1}{2}} = \left(\frac{\dot{u}\dot{x}}{\dot{x}^2 + \dot{y}^2} + \dot{z}^2\right)^{\frac{1}{2}}$ the fluxion of the furface will be $lh \times \sqrt{(\dot{x}^2 + \dot{y}^2)}$. From the given equation expreffing the relation between the two abfciffæ z and x and ordinates y find, by affuming z invariable $p\dot{x} = \dot{y}$, and by affuming x invariable $q\dot{z} = \dot{y}' = \dot{u}$,

$\dot{y}' = \dot{u}$, which being substituted for their values in the quantity $lh \times \sqrt{(\dot{x}^2 + \dot{y}^2)}$, there will result $(q^2 + p^2 + 1)^{\frac{1}{2}} \times \dot{x} \times \dot{z} = A\dot{x}\dot{z} = \frac{(q^2 + p^2 + 1)^{\frac{1}{2}}}{p} \times \dot{y} \times \dot{z} = B\dot{y}\dot{z}$; in A and B for y and x respectively substitute their value deduced from the given equation, and let the resulting quantities be $A'\dot{x}\dot{z}$ and $B'\dot{y}\dot{z}$, where A' is a function of x and z, and B' a function of y and z; find the fluent of $A'\dot{x}\dot{z}$ from the supposition that x is only variable contained between the extreme values of x to a given value of z, which let be $L\dot{z}$, then find the fluent of $L\dot{z}$ by supposing z only variable contained between given values of z, and it will be the surface of the solid contained between those values.

The same may be deduced by finding the fluent of $B'\dot{y}\dot{z}$ on the supposition that y is the only variable quantity contained between the extreme values of y as before of x to a given value of z, which let be $L'\dot{z}$; then will the fluent of $L'\dot{z}$ contained between the given values of z be the surface required.

If the solid be a cone generated by the rotation of a rectangular triangle round a side containing the right angle as an axis; hu will be a given quantity, if $\dot{z}$ be given.

If the above-mentioned angles are given, but not right ones, the arc $p'm$ and perpendicular $l'h$ can easily be deduced, and consequently the increment of the surface.

3. To define a curve of double curvature, it is necessary to have two equations expressing the relation between the abscissæ z and x and their ordinates (y) given, and if the angles which they respectively make with each other be right ones; the fluxion of the arc as given in the *Proprietates Curvarum* is $(\dot{z}^2 + \dot{x}^2 + \dot{y}^2)^{\frac{1}{2}}$. Find its value from the two given equations in terms of x, y, or z, multiplied into its respective fluxions, and its fluent, properly corrected, will be the length of the arc required.

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If the angles are not right, they may eaſily be reduced to them.

4. The attractions of theſe ſurfaces, curves, &c. on a given point P may be deduced from the preceding principles of finding the attractions of each of the parts in the directions of the firſt abſciſſa, which paſſes through the point P, the ſecond abſciſſa, and the ordinates, and then finding the integrals of theſe increments.

From the method which determines the attraction of a body, ſurface, &c. on a given point can be determined the attraction of a body, &c. on any number of points, and conſequently the attraction of one body, &c. on another, &c.

It is ſometimes advantageous to transform the firſt abſciſs, that it may paſs through the point attracted; and the abſciſſæ and ordinates, that they may be at right angles to each other, &c.

PROBLEM V.

1. Fig. 8. Given an equation expreſſing the relation between the two abſciſſæ AP and Pp of a ſolid, and their correſpondent ordinates pm, or AP′, P′p', and $p'm'$; to transform the firſt abſciſſa into any other Lh.

Let the abſciſſa Lh begin from a point L of the firſt abſciſſa AP, and meet an ordinate pm in the point h; draw hp, and let the ſines of the angles Ppm, Php, and pPh; LPh, PhL, and PLh, be denoted reſpectively by r, s, and t, &c. r', s', and t'; through a point h of the line Ph draw $p'h'm'$ parallel to pm, and $Lh = z$, $hh' = x$, and $h'm' = y$: in the given equation for AP, Pp, and pm ſubſtitute reſpectively their correſpondent

respondent values $\frac{s'z}{r'} \pm AL(a)$, $\frac{st'z}{rr'} \pm \frac{sx}{r}$ (for $Pb = \frac{t'z}{r'}$ and $Pb' = Pb \pm bb' = \frac{t'z}{r'} \pm x$), and $y \pm \frac{tt'z}{rr'} \pm \frac{tx}{r'}$; there results an equation to the same solid expressing the relation between the two abscissæ $z = Lb$ and x, and their correspondent ordinates y.

1. 2. If the abscis Lb does not begin from L, a point in the first given abscis AP, but from M a point given out of it, it may be reduced to the preceding case, by drawing from M a line $MN = c$ to the plane of the first and second abscissæ parallel to the ordinates pm; and from N to the first abscissa a line $NO = b$ parallel to the second abscissæ, and substituting in the equation expressing the relation between AP, Pp, and pm for AP, Pp, and pm respectively $z \pm AO$ (a), $x \pm b$ and $y \pm c$; and there results the equation required expressing the relation between the two abscissæ z and x, and their correspondent ordinates y, of which the first abscissa z passes through the point M.

2. To change the second abscissa Pp into any other Lb, the first abscissa and ordinates remaining the same. In the preceding figure let L be considered as a moveable point of the first abscis AL, and the sines of the respective angles denoted by the same letters as before, and $Lb = x$, $AL = z$, and $bm = y$; in the given equation for AP, Pp, and pm, substitute $z \pm \frac{s'x}{r'}$, $\frac{st'x}{rr'}$, and $y \pm \frac{tt'x}{rr'}$; and there will result the equation required expressing the relation between z and x the abscissæ, and their correspondent ordinates y.

3. Fig 8. To change the ordinates, the abscissæ remaining the same, draw $p'm$ an ordinate transformed, $p'b$ parallel to the first abscissa AP, and meeting a second abscissa, of which

 pm

pm is an ordinate in h: for the fines of the angles $p'hp$, hpp', and $hp'p$; $p'pm$, pmp', and $pp'm$ write r, s, and t, r', s', and t'; and for AP′, P′p', and $p'm$ refpectively z, x, and y; then fubftitute in the given equation for AP, Pp, and pm, their refpective values z $(AP') \pm \frac{ss'}{rr'} \times y$, x $(P'p' \pm \frac{ts'}{rr'} y$, and $\frac{t'}{r'} y$; and there refults an equation to the folid expreffing the relation between the two abfciffæ AP′ and P′p' and the tranformed ordinates $p'm$.

From thefe cafes, which are eafily reducible to one, may be transformed any given abfciffæ and their correfpondent ordinates into any other containing given angles, &c. with the before-mentioned abfciffæ and ordinates.

In the properties of curve lines, firft publifhed in 1762, is given a method of deducing the equation to any fection of the folid, and in particular the cafe of deducing the equation to the projection of any curve on a given plane.

From the principles given in this, and the Paper on centripetal forces, which the Royal Society did me the honour to print, can be deduced the fluxional equations, whofe fluents exprefs the relations between the abfciffæ and their correfpondent ordinates of the curves defcribed by bodies, of which the particles act on each other with forces varying according to given functions of their diftances.

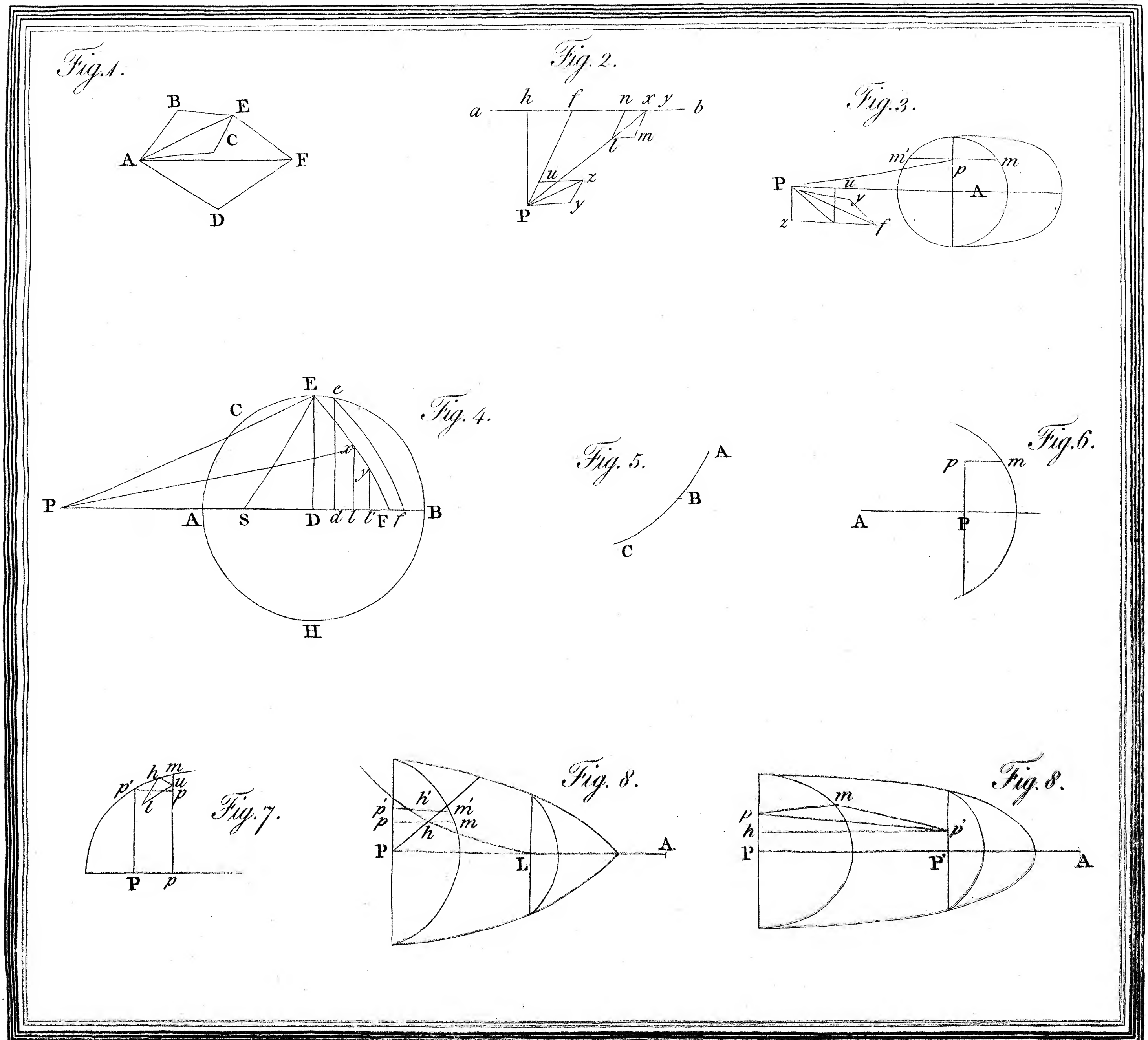
Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.
Fig. 5.
Fig. 6.
Fig. 7.
Fig. 8.
Fig. 8.